A Differential Method of Reflection Coefficient Extraction From FDTD Simulations

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Abstract— **A novel concept for accurate extraction of the reflection coefficient from electromagnetic simulations is proposed and implemented into the finite-difference time-domain (FDTD) method. It uses only the values of an arbitrary tangential E-field component and its derivative with respect to the longitudinal direction at one selected point, and of an arbitrary tangential H-field component and its derivative with respect to the longitudinal direction at another point. No *a priori* knowledge of reference impedances is needed, and the reflection coefficient is extracted from a single run of FDTD. The proposed method is directly applicable to arbitrarily shaped and inhomogeneous transmission lines.**

I. Introduction

ACCURATE extraction of reflection coefficient (denoted in this letter by Γ) from finite-difference time-domain (FDTD) simulations is an important practical problem. The value of Γ is used to obtain the S parameters of the analysed circuit. A common approach cosists of matching the output(s) by means of high-quality absorbing boundary conditions (ABC's), which permits us to assume S_{11} to be equal to Γ at the input. Recently, a more elaborate method has been proposed [8] in which multiple calculations of reflection coefficient at different ports allow us to obtain the S-parameters with good accuracy, even for relatively poor ABC's. In either case, the main problem of Γ extraction resides in separating the incident and reflected waves at the considered port. To solve this problem, two basic approaches have been reported so far.

The first approach [1]–[3] consists of running the simulation twice (or concurrently on two circuits): one run is for the analyzed circuit and the other for a nonreflecting structure with the identical input. The second simulation provides a pure incident wave. It can be reduced to the two-dimensional (2-D) simulation of the cross section of the line [8], but at the expense of repeating it independently for each relevant frequency. Although good results in applying this approach have been reported, its clear disadvantage is the necessity of double simulation, which significantly increases the employed computer resources and complicates the use of the program.

The second approach [4], [5] assumes that for reference, either wave impedance or characteristic impedance of the line is *a priori* known (analytically or heuristically). After application of correction factors due to the shifts between the E and H fields in time and space, this method gives very good results for homogeneously-filled TEM lines [4]–[6] (when the wave impedance does not change with frequency) or waveguides [7] (when it changes according to an analytically known rule). However, in application to inhomogeneous quasi-TEM lines, even if we know the quasi-static wave impedance, significant errors appear due to dispersive properties that are difficult to predict.

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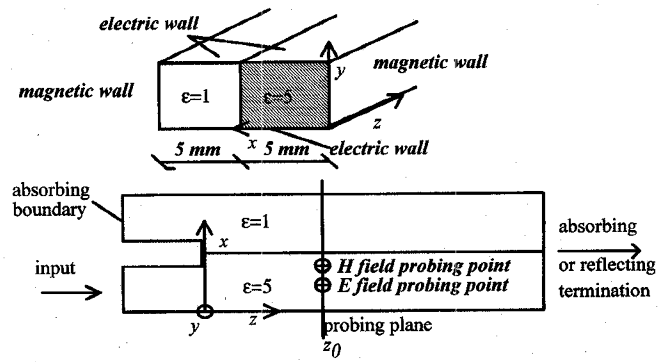

Fig. 1. The structure used to verify accuracy of the present method.

II. The New Approach

Let us assume that the wave in the line propagates in the z direction. The new approach is based on the following observation: For calculation of Γ, we really do not need to know either the wave impedance or the characteristic impedance of the input line. Having assumed a monomode transmission in the plane z_0 of the considered port, we can calculate Γ entirely from the components $E_t(x_e, y_e, z_0)$ and $H_t(x_h, y_h, z_0)$ and their space derivatives with respect to the direction of propagation.

Please note that in most applications we consider the reflection coefficient for the dominant mode in the frequency range in which other modes are evanescent. With the reference plane sufficiently separated from the line discontinuities, the assumption of monomode transmission is well justified.

Our method is directly applicable to arbitrarily shaped lines. For tutorial purposes, in this letter we shall refer to an inhomogeneously filled parallel-plate waveguide (Fig. 1). It is a quasi-TEM line with general properties comparable with practical lines of this class such as microstrip or coplanar line in the sense that its wave impedance depends on both the probing point and the frequency. On the other hand, it is a relatively simple structure with easily predictable higher waveguide modes.

For a particular frequency ω, the chosen total field components can be described by

$$E_t(x_e, y_e, z) = E_t^+(x_e, y_e, z_0) \times \left(e^{-j\beta(z-z_0)} + \Gamma(z_0)e^{j\beta(z-z_0)}\right) \quad (1)$$

$$H_t(x_h, y_h, z) = H_t^+(x_h, y_h, z_0) \times \left(e^{-j\beta(z-z_0)} - \Gamma(z_0)e^{j\beta(z-z_0)}\right) \quad (2)$$

where E_t^+ and H_t^+ are the incident wave components.

Let us now express the derivatives of H_t and E_t with respect to z

$$\frac{\partial E_t(x_e, y_e, z)}{\partial z} = -j\beta E_t^+(x_e, y_e, z_0) \times \left(e^{-j\beta(z-z_0)} - \Gamma(z_0)e^{j\beta(z-z_0)}\right) \quad (3)$$

$$\frac{\partial H_t(x_h, y_h, z)}{\partial z} = -j\beta H_t^+(x_h, y_h, z_0) \times \left(e^{-j\beta(z-z_0)} + \Gamma(z_0)e^{j\beta(z-z_0)}\right). \quad (4)$$

Using (1)–(4) we obtain the expressions for Γ and β

$$\Gamma = (A-1)/(A+1) \quad (5)$$

with

$$A = \pm\text{sqrt}\left[\frac{E_t(x_e, y_e, z)\frac{\partial H_t(x_h, y_h, z)}{\partial z}}{H_t(x_h, y_h, z)\frac{\partial E_t(x_e, y_e, z)}{\partial z}}\right]$$
$$\beta = \pm\text{sqrt}\left[\frac{\frac{\partial E_t(x_e, y_e, z)}{\partial z}\frac{\partial H_t(x_h, y_h, z)}{\partial z}}{H_t(x_h, y_h, z)E_t(x_e, y_e, z)}\right]. \quad (6)$$

Ambiguity of sign in (5) and (6) can be eliminated assuming that $|\Gamma| \leq 1$ and $\text{Re}(\beta)$ is positive.

III. FDTD Implementation

In the FDTD algorithm we must take into account the time and space offsets between the particular components of E and H. Since the position of the chosen components in the xy plane is arbitrary, the offsets in this plane are irrelevant. Let us assume that the E-field components are available at the time instants $t = n\Delta t$ and in the planes $z = ka + z_0$ (where a is the cell size and k is an integer), while the H-field components are available at the time instants $t = (n+0.5)\Delta t$ and in the planes $z = (k+0.5)a + z_0$. Assuming that we set our probing plane to $z = z_0$ we calculate

$$E_t(z_0, \omega) = \mathbf{F}\{E_t(z_0, t)\} \quad (7)$$

$$\frac{\partial E_t(z_0, \omega)}{\partial z} = \mathbf{F}\{0.5(E_t(z_0+a, t) - E_t(z_0-a, t))/a\} \quad (8)$$

$$H_t(z_0, \omega) = \mathbf{F}\{0.5(H_t(z_0+a/2, t+\Delta t/2) + H_t(z_0-a/2, t+\Delta t/2))\} \times \exp(-j\omega\Delta t/2) \quad (9)$$

$$\frac{\partial H_t(z_0, \omega)}{\partial z} = \mathbf{F}\{(H_t(z_0+a/2, t+\Delta t/2) - H_t(z_0-a/2, t+\Delta t/2)/a)\} \times \exp(-j\omega\Delta t/2) \quad (10)$$

where $\mathbf{F}$ denotes a Fourier transform.

Equations (7)–(10) can serve to calculate Γ directly from (5). However, there also exists a slightly more elaborate way that permits to reduce the error of discrete field approximation in the z direction. First, the value of β is extracted using (6)–(10) and then Γ is calculated from (5) and (7)–(10), but with

—derivative of electric field (8) further divided by $\sin(\beta a)/(\beta a)$;

—magnetic field of (9) divided by $\cos(\beta a/2)$;

—derivative of magnetic field of (10) divided by $\sin(\beta a/2)/(\beta a/2)$.

The role of the above correction factors resides in enforcing the sinusoidal field variation along the line, between the neighbouring nodes considered in (7)–(10). Without these corrections, direct use of (7)–(10) would amount to the assumption of piece-wise linear field variation between the nodes.

IV. Examples of Application

Let us consider a structure presented in Fig. 1 that is based on the parallel-plate waveguide. The input of the line has been arranged in such a way as to generate in the plane $z = 0$ the dominant quasi-TEM mode as well as a series of higher waveguide modes with amplitudes comparable to the dominant one. Such an input has been chosen to validate the method under extreemely unfavourable conditions. The first waveguide mode (which will be called TE1) has the cutoff frequency of about 10.4 GHz. We will consider extracting Γ for the dominant quasi-TEM mode in the band from 0.1–9.4 GHz. In this band all the waveguide modes are evanescent and their amplitudes decrease as we move away from the plane $z = 0$. By applying the (5)–(10) in different probing planes $z = z_0$, we shall investigate the influence of the higher modes' amplitudes on the accuracy of our method.

As the first example we will consider a matched line that should give $\Gamma = 0$. The FDTD algorithm with cell size $a = 1$ mm has been used. To avoid any influence of the absorbing boundary conditions, the line was made long enough to terminate the computing before the reflections from the far end reach the probing plane.

In principle, the probing points for E_t and H_t can be chosen arbitrarily. However, the most reasonable choice is where we expect high amplitude of the dominant mode and relatively low amplitude of the higher modes. In the case of Fig. 1 we can predict that the TE1 mode has a minimum close to the center of the line but somewhat shifted towards the dielectric. Thus, we choose E_t being the y component at $x_e = 3.5$ mm and H_t being the x component at $x_h = 4.5$ mm. The position along the y axis is irrelevant due to the 2-D properties of the structure. Fig. 2 presents Γ versus frequency calculated for $z_0 = 10$, 20, and 30 mm.

Now, we will consider an example permitting us to compare the new approach to the previous ones. Let us assume that we want to calculate the reflection coefficient for a segment of the line of Fig. 1, but with a tapered dielectric and terminated by ABC's. Fig. 3 presents the view of the line and results of calculations in three cases.

In case **a** the reference wave impedance (defined as $Z^+ = E_t^+(x_e, y_e, z_0)/H_t^+(x_h, y_h, z_0)$) has been obtained from an

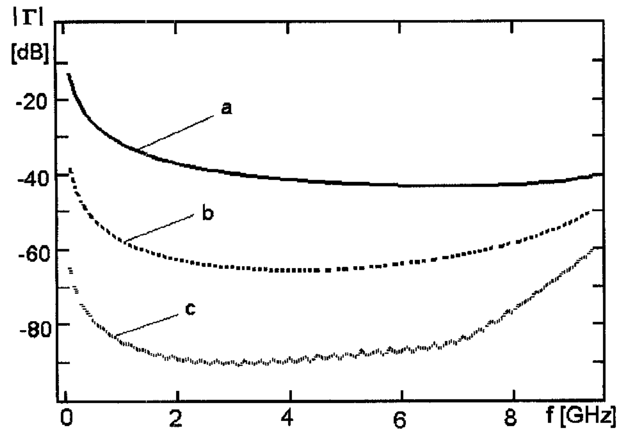

Fig. 2. Calculated $|\Gamma|$ versus frequency in the structure of Fig. 1 with absorbing termination and $x_e = 2.5$ mm, $x_h = 7.5$ mm and a—$z_0 = 10$ mm, b—$z_0 = 20$ mm, c—$z_0 = 30$ mm.

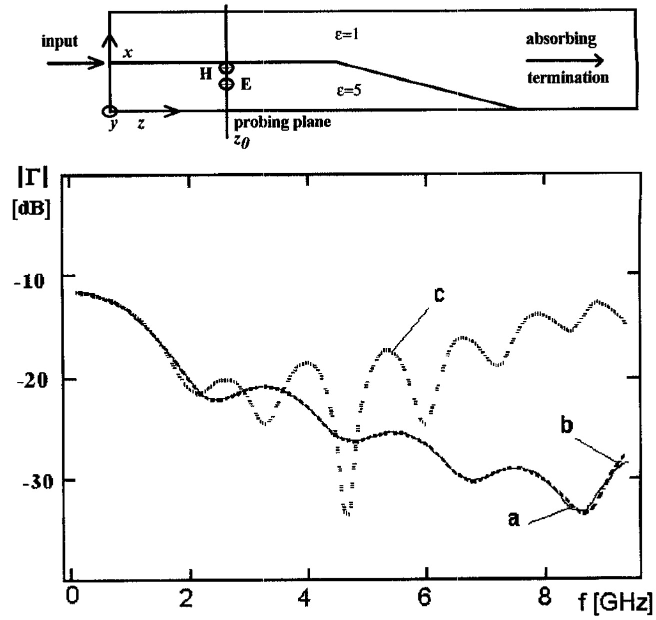

Fig. 3. Calculated $|\Gamma|$ versus frequency in the structure presented above assuming $z_0 = 20$ mm and the tapered section situated between $z_1 = 45$ mm and $z_2 = 80$ mm: **a**—with a separate run for calculation of reterence impedance, **b**—by the present method, c—assuming the quasi-static impedance as reference.

additional simulation in a very long transmission line. The reflection coefficient was then calculated from the equation $\Gamma = (Z - Z^+)/(Z + Z^+)$ with $Z = E_t(x_e, y_e, z_0)/H_t(x_h, y_h, z_0)$ obtained in the simulation of the circuit of Fig. 3. In case **b** we have calculated Γ by the new differential method. In case **c** no reference simulations have been performed and the analytically calculated quasi-static wave impedance has been used as a reference for Γ extraction. It should be noted that the new method gives practically the same results as the method of case **a**. However, its great advantage is that the results have been obtained in a single run with no laborious reference simulations. On the other hand, it can be seen that when the reference simulations are avoided by applying the analytically known quasistatic wave impedance (case **c**), completely wrong results are obtained for the considered dispersive structure.

The properties of the new method have been presented here on a relatively simple example of the inhomogeneous parallel-plate waveguide, but it is directly applicable to various practical cases including microstrip lines as well as rectangular and circular waveguides. Moreover, in this letter we have only discussed input Γ calculations. The method can be applied simultaneously to calculations of Γ at the input and output(s), thus it can be envisaged as a part of a universal system for S-parameter extraction similar, for example, to the one presented in [8].[1]

V. Conclusion

The proposed method for reflection coefficient extraction proved to be simple, universal, and very accurate providing that monomode transmission is assured in the considered probing plane. Contamination of the dominant mode by higher modes deteriorates the accuracy, but this effect can be reduced by a proper choice of points where the fields are considered. The authors believe that further research will bring better ways to improve the accuracy of the method in the presence of higher modes and will confirm its applicability in the systems of multiport S-parameter extraction.

[1] It has been brought to the authors' attention by one of the reviewers that a principle similar to the one used by them has been independently developed in the SCN TLM method. While it has not been published in the open literature, an outline of its TLM application can be found in the user's manual of Micro-Stripes simulator of KCC Consultants Ltd. of Nottingham, U.K., June 1995.

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